

Contents

Part I An Introduction to Lévy and Feller Processes

Preface	3
Symbols and Notation	5
1 Orientation	7
2 Lévy Processes	13
3 Examples	17
4 On the Markov Property	27
5 A Digression: Semigroups	35
6 The Generator of a Lévy Process	41
7 Construction of Lévy Processes	49
8 Two Special Lévy Processes	55
9 Random Measures	63
10 A Digression: Stochastic Integrals	73
11 From Lévy to Feller Processes	87
12 Symbols and Semimartingales	99
13 Dénouement	109
Appendix: Some Classical Results	115
Bibliography	123

Part II Invariance and Comparison Principles for Parabolic Stochastic Partial Differential Equations

Preface	129
14 White Noise	
14.1 Some heuristics	133
14.2 LCA groups	134
14.3 White noise on G	136
14.4 Space-time white noise	139
14.5 The Walsh stochastic integral	140
14.5.1 Simple random fields	141
14.5.2 Elementary random fields	142
14.5.3 Walsh-integrable random fields	142
14.6 Moment inequalities	144
14.7 Examples of Walsh-integrable random fields	145
14.7.1 Integral kernels	145
14.7.2 Stochastic convolutions	147
14.7.3 Relation to Itô integrals	148
15 Lévy Processes	
15.1 Introduction	151
15.1.1 Lévy processes on \mathbb{R}	152
15.1.2 Lévy processes on \mathbb{T}	155
15.1.3 Lévy processes on \mathbb{Z}	156
15.1.4 Lévy processes on $\mathbb{Z}/n\mathbb{Z}$	156
15.2 The semigroup	157
15.3 The Kolmogorov–Fokker–Planck equation	158
15.3.1 Lévy processes on \mathbb{R}	158
16 SPDEs	
16.1 A heat equation	167
16.2 A parabolic SPDE	169
16.2.1 Lévy processes on \mathbb{R}	171
16.2.2 Lévy processes on a denumerable LCA group	172
16.2.3 Proof of Theorem 16.2.2	173
16.3 Examples	175
16.3.1 The trivial group	175
16.3.2 The cyclic group on two elements	176
16.3.3 The integer group	178
16.3.4 The additive reals	179
16.3.5 Higher dimensions	180

17 An Invariance Principle for Parabolic SPDEs	
17.1 A central limit theorem	183
17.2 A local central limit theorem	185
17.3 Particle systems	190
18 Comparison Theorems	
18.1 Positivity	195
18.2 The Cox–Fleischmann–Greven inequality	196
18.3 Slepian’s inequality	199
19 A Dash of Color	
19.1 Reproducing kernel Hilbert spaces	203
19.2 Colored noise	205
19.2.1 Example: white noise	206
19.2.2 Example: Hilbert–Schmidt covariance	206
19.2.3 Example: spatially-homogeneous covariance	207
19.2.4 Example: tensor-product covariance	208
19.3 Linear SPDEs with colored-noise forcing	210
Bibliography	213
Index	217