

Index

- absorbing point, 100
- adapted, 27, 119
- almost positive (PP), 46

- Blumenthal–Gettoor–Pruitt index, 112
- bounded coefficients, 92
- Brownian motion, 18, 153
 - characterization among LP, 57
 - construction, 58–61
 - generator, 43, 161
 - martingale characterization, 57
 - white noise, 74
- Burkholder–Davis–Gundy inequality, 145

- $C_c(G)$, 141
- Campbell’s formula, 18–19
- canonical filtration, 13
- Cauchy–Abel functional equation, 14, 115
- Central limit theorem, 184
 - local version, 187
- Chapman–Kolmogorov equations, 27
- characteristic exponent, 16, 152
 - is polynomially bounded, 42
 - of a stable process, 23
 - is subadditive, 42
- characteristic operator, 101
- closed operator, 37
- colored noise, 204
- compound Poisson process, 18, 49, 153
 - building block of LP, 52, 69
 - distribution, 20
 - is a Lévy process, 19
- continuous in probability, 7, 14
- control measure, 73
- convergence of types theorem, 117
- convolution semigroup, 28, 29
- correlation function, 75

- Courrège–von Waldenfels theorem, 89
- covariance operator, 204
- Cox–Fleischmann–Greven inequality, 197
- cyclic groups, 135

- de Finetti’s theorem, 8
- dissipative operator, 39
- Dynkin’s formula, 100
- Dynkin’s operator, 101
- Dynkin–Reuter lemma, 40

- elementary random field, 142
- exponential martingale, 67

- Feller process, 32, 87
 - asymptotics as $t \rightarrow 0$, 112
 - càdlàg modification, 87
 - conservativeness, 95
 - generator, 36, 40, 89
 - Hausdorff dimension, 112
 - maximal estimate, 110, 111
 - moments, 111
 - semimartingale characteristics, 103
 - strong Markov property, 88
- Feller property, 30, 35
- Feller semigroup, 32, 35
 - extension to $\mathcal{B}_b(\mathbb{R}^d)$, 35
- Feller semigroup, *see also* Feller process
- Fokker–Planck equation, 158, 160
- Fourier transform, 41

- Gamma process, 31
- Gaussian interpolation, 199
- generator, 36
 - of Brownian motion, 43, 161
 - domain, 40, 42
 - Dynkin’s operator, 101

- of a Feller process, 89
 - Hille–Yosida–Ray theorem, 39
 - of a Lévy process, 44
 - of a Lévy process on \mathbb{Z} , 163
 - of a Lévy process on $\mathbb{Z}/2\mathbb{Z}$, 164
 - no proper extension, 40
 - positive maximum principle, 39
 - stable process, 162
 - uniform motion, 161
- group topology, 134
- Haar measure, 135
- Hilbert–Schmidt operators, 206
- Hille–Yosida–Ray theorem, 39
- homomorphism, 134
- independent increments (L2), 7
- index of self-similarity, 21
- index of stability, 23
- infinitely divisible, 8
- infinitesimal generator, *see* generator
- intensity measure, 65
- Itô isometry, 79, 80
- Itô’s formula, 85
- jump measure, 63
- Kolmogorov’s equation, 158, 160
- (L0), (L1), (L2), (L3), 7
- $L^1_{\text{loc}}(G)$, 203
- (L2’), 13
- $L^2_{\mathbb{K}}(G)$, 205
- Lévy measure, 48
- Lévy process, 13, 151
 - α -potential, 171
 - càdlàg modification, 14, 87
 - characteristic exponent, 16
 - characterization, 17
 - diffusion coefficient, 152
 - drift coefficient, 152
 - finite-dimensional distributions, 15
 - generator, 44
 - independent components, 67
 - infinite divisibility, 8, 13, 16
 - infinite life-time, 48
 - intensity measure, 65
 - jump measure, 63
 - Lévy measure, 152
 - limit of LPs, 51
 - Markov property, 29
 - moments, 24, 56
 - no fixed discontinuities, 15
 - stable process, 22, 23, 154
 - generator, 162
 - starting from x , 28
 - strong Markov property, 32
 - sum of two LP, 50
 - symbol, 44
 - tempered stable, 172
 - transition function, 168
 - translation invariance, 28, 44
- Lévy triplet, 48
- Lévy’s continuity theorem, 117
- Lévy’s theorem, 57
- Lévy–Itô decomposition, 9, 53, 70, 83
- Lévy–Khintchine formula, 8, 25, 44, 72, 152
- Lévy-type process, 109
- LCA group, 134
- linear heat equation, 167
- Markov process, 27, 28
 - strong Markov property, 32, 88
 - universal Markov process, 28
- martingale noise, 75
- mild solution, 170
- orthogonal increments, 75
- Picard iteration sequence, 173
- PMP (positive maximum principle), 39
- Poisson process, 18
 - characterization among LP, 55
 - is a Lévy process, 19
 - jump measure of LP, 64, 68
- Poisson process, *see also* compound –
- Poisson random measure, 76
 - compensator, 84
- positive definite, 203
- positive maximum principle (PMP), 39

- positivity principle
 - for SDEs, 195
 - for SPDEs, 196
- PP (almost positive), 46
- predictable, 119
 - process, 148
- pseudo-differential operator, 44, 89
- random linear functional, 138, 204
- random orthogonal measure, 73
 - stochastic integral, 77, 79
- reproducing kernel Hilbert space, 205
- resolvent, 37
- second-order process, 75
- self-similar process, 23
- semigroup, 29, 157
 - conservative, 30
 - contractive, 30, 39
 - convolution, 157
 - Feller property, 30, 35
 - strong Feller property, 30
 - strongly continuous, 30, 39
 - sub-Markovian, 30
- semimartingale, 84, 103
- simple random field, 141
- Slepian's inequality, 199
- space-time noise, 77, 79
- SPDE, 167
 - connection with 1-dimensional SDEs, 175
 - connection with 2-dimensional SDEs, 176
 - existence and uniqueness result, 170
 - in dimension > 1 , 180
 - in higher dimensions, 181
 - and interacting diffusions, 179
 - on \mathbb{R} , 179
 - on the trivial group, 175
 - on \mathbb{Z} , 178
 - on $\mathbb{Z}/n\mathbb{Z}$, 176
- stable process, 22
- stable random variable, 22
- stable-like process, 109
- stationary increments (L1), 7
- stochastic convolution, 148
- stochastic integral, 77, 79
- stochastic interval, 80, 120
- strong continuity, 30, 39
- strong Feller property, 30
 - of a Lévy process, 31
- strong Markov property
 - of a Feller process, 88
 - of a Lévy process, 32
- symbol, 43, 44
 - bounded coefficients, 92
 - continuous in x , 93, 95
 - of a Feller process, 89
 - of a Lévy process, 44
 - of an SDE, 105
 - probabilistic formula, 99, 102
 - sector condition, 111
 - upper semicontinuous in x , 95
- temporal support, 141
- tensor product, 208
- topological group, 134
- transition function, 27
- transition semigroup, *see* semigroup
- translation invariant, 28, 44, 118
- trivial group, 134
- uniform motion, 152
 - generator, 161
- vague convergence, 116
- Walsh integral, 142
 - relation to the Itô integral, 143, 148
- Walsh isometry, 142
- Walsh-integrable random fields, 143
- white noise, 74, 77, 136
 - is not σ -additive, 74
 - space-time, 139
- Wiener integral, 138
 - and Brownian motion, 139
- Wiener isometry, **136**, 138

Symbols and Notation

This index is intended to aid cross-referencing, so notation that is specific to a single chapter is generally not listed. Some symbols are used locally, without ambiguity, in senses other than those given below; numbers following an entry are page numbers.

Unless otherwise stated, functions are real-valued and binary operations between functions such as $f \pm g$, $f \cdot g$, $f \wedge g$, $f \vee g$, comparisons $f \leq g$, $f < g$ or limiting relations $f_n \xrightarrow{n \rightarrow \infty} f$, $\lim_n f_n$, $\liminf_n f_n$, $\limsup_n f_n$, $\sup_n f_n$ or $\inf_n f_n$ are understood pointwise.

General notation: analysis

positive	always in the sense ≥ 0
negative	always in the sense ≤ 0
\mathbb{N}	$1, 2, 3, \dots$
$\inf \emptyset$	$\inf \emptyset = +\infty$
$a \vee b$	maximum of a and b
$a \wedge b$	minimum of a and b
$[x]$	largest integer $n \leq x$
$ x $	norm in \mathbb{R}^d : $ x ^2 = x_1^2 + \dots + x_d^2$
$x \cdot y$	scalar product in \mathbb{R}^d : $\sum_{j=1}^d x_j y_j$
$\mathbb{1}_A$	$\mathbb{1}_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$
δ_x	point mass at x
\mathcal{D}	domain
Δ	Laplace operator
∂_j	partial derivative $\frac{\partial}{\partial x_j}$
∇, ∇_x	gradient $(\frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_d})^\top$
$\mathcal{F}f, \hat{f}$	Fourier transform $(2\pi)^{-d} \int e^{-ix \cdot \xi} f(x) dx$
$\mathcal{F}^{-1}f, \check{f}$	inverse Fourier transform

$$\int e^{ix \cdot \xi} f(x) dx$$

$$e_\xi(x) = e^{-ix \cdot \xi}$$

General notation: probability

\sim	'is distributed as'
\perp	'is stochastically independent'
a.s.	almost surely (w. r. t. \mathbb{P})
iid	independent and identically distributed
$\mathbb{N}, \text{Exp}, \text{Poi}$	normal, exponential, Poisson distribution
\mathbb{P}, \mathbb{E}	probability, expectation
\mathbb{V}, Cov	variance, covariance
(L0)–(L3)	definition of a Lévy process, 7
(L2')	13

Sets and σ -algebras

A^c	complement of the set A
\overline{A}	closure of the set A
$A \cup B$	disjoint union, i.e., $A \cup B$ for disjoint sets $A \cap B = \emptyset$
$B_r(x)$	open ball, centre x , radius r

$\text{supp } f$	support, $\overline{\{f \neq 0\}}$
$\mathcal{B}(E)$	Borel sets of E
\mathcal{F}_t^X	canonical filtration $\sigma(X_s : s \leq t)$
\mathcal{F}_∞	$\sigma\left(\bigcup_{t \geq 0} \mathcal{F}_t\right)$
\mathcal{F}_τ	88
$\mathcal{F}_{\tau+}$	32
\mathcal{P}	predictable σ -algebra, 119

Stochastic processes

$\mathbb{P}^x, \mathbb{E}^x$	law and mean of a Markov process starting at x , 28
X_{t-}	left limit $\lim_{s \uparrow t} X_s$
ΔX_t	jump at time t : $X_t - X_{t-}$
σ, τ	stopping times: $\{\sigma \leq t\} \in \mathcal{F}_t, t \geq 0$
τ_r^x, τ_r	$\inf\{t > 0 : X_t - X_0 \geq r\}$, first exit time from the open ball $B_r(x)$ centered at $x = X_0$
càdlàg	right continuous on $[0, \infty)$ with finite left limits on $(0, \infty)$

Spaces of functions

$\mathcal{B}(E)$	Borel functions on E
$\mathcal{B}_b(E)$	- - , bounded
$\mathcal{C}(E)$	continuous functions on E
$\mathcal{C}_b(E)$	- - , bounded
$\mathcal{C}_\infty(E)$	- - , $\lim_{ x \rightarrow \infty} f(x) = 0$
$\mathcal{C}_c(E)$	- - , compact support
$\mathcal{C}^n(E)$	n times continuously diff'ble functions on E
$\mathcal{C}_b^n(E)$	- - , bounded (with all derivatives)
$\mathcal{C}_\infty^n(E)$	- - , 0 at infinity (with all derivatives)
$\mathcal{C}_c^n(E)$	- - , compact support
$L^p(E, \mu), L^p(\mu), L^p(E)$	L^p space w. r. t. the measure space (E, \mathcal{A}, μ)
$\mathcal{S}(\mathbb{R}^d)$	rapidly decreasing smooth functions on \mathbb{R}^d , 41