

Index

- additive functional 382
- analytic function of bounded type 85
 - canonical factorization 85
- angular limit 76, 90

- Balakrishnan's formula 217
- Bernstein function 21, 202
 - characterization 27, 212
 - complete
 - see* complete Bernstein function
 - cone structure 28
 - convergence of 29
 - definition 21
 - extended 52, 87, 127
 - extremal representation 33
 - holomorphic on $\vec{\mathbb{H}}$ 25
 - is negative definite 38, 41
 - Lévy measure 22
 - Lévy triplet 22
 - Lévy–Khintchine representation 22,
51, 58, 65, 69, 184, 205, 269
 - preserves sectors 25
 - relation with \mathcal{CM} 27, 49, 53
 - relative compactness 7
 - special
 - see* special Bernstein function
 - stability 28
 - subclass \mathcal{PBF} 61, 139
 - tail behaviour 24
 - transformation
 - see* transformation of \mathcal{BF}
- Bernstein's theorem 3, 21, 40, 70
- Bochner's theorem 46
- Bochner–Schoenberg theorem 212
- Bondesson class 117
 - and first passage time 289
 - as vague closure 120
 - definition 117
 - stability 117

- $\text{BO} \subset \text{ID}$ 117
- $\text{BO} \not\subset \text{SD}$ 128

- capacity 243, 381
- Choquet representation
 - for \mathcal{BF} 33
 - for \mathcal{CBF} 95
 - for \mathcal{CM} 8
 - for \mathcal{S} 18
 - for \mathcal{JBF} 114
 - for positive definite fns 40–41
- complete Bernstein function 69
 - $\mathcal{CBF} - \mathcal{S}$ 80
 - and inverse local time 270
 - characterization 69–70, 92, 93, 96, 99,
181
 - composition with \mathcal{H} 176
 - cone structure 94
 - conjugate pair 92
 - convergence of 95
 - definition 69
 - exponential of 87
 - exponential representation 86, 99,
100, 119, 124, 366
 - extended
 - see* extended \mathcal{CBF}
 - extremal representation 95
 - is Nevanlinna–Pick function 70
 - is operator monotone 190
 - is Pick function 70
 - kernel representation 108
 - Lévy vs. Stieltjes measure 80, 246
 - log-convexity of \mathcal{CBF} 99, 100, 102
 - logarithm of 87
 - multiplicative representation 87
 - Nevanlinna representation 78, 102
 - potential density 162
 - relation with \mathcal{S} 93, 96, 99
 - stability 93, 94, 98, 118
 - Stieltjes measure 75

- Stieltjes representation 75, 92, 93, 95, 101, 117, 181, 185
- symmetric 103, 105
- $\mathcal{CBF} \subset \mathcal{SBF}$ 159
- completely monotone function 2
 - compl. self-decomposable 60
 - cone structure 5
 - convergence of 6
 - definition 2
 - extremal representation 8
 - infinitely divisible 51
 - is positive definite 38, 41
 - iterated differences 40
 - logarithmically
 - see* log-completely monotone
 - on \mathbb{R} 13
 - p -times self-decomposable 59
 - representation 3
 - self-decomposable 55
 - stability 5, 28, 29, 58, 376
 - stable 55
- contractivity
 - hypercontractivity 241
 - supercontractivity 241
 - ultracontractivity 233, 241
- convolution of exp. distributions 125
 - and first passage time 289, 293
 - $CE \subset GGC$ 125
- convolution semigroup 48, 202, 205
 - characterization 49
 - definition 48
 - potential measure of 63
 - subordinate 65
- Dirichlet form 243, 380
- dissipative operator 180, 201
 - spectrum 180
- distribution
 - \mathcal{S} -self-decomposable 123
 - infinitely divisible 51, 54
 - Laplace exponent of 128
 - overview 127
 - relations among 128
 - self-decomposable
 - see* self-decomposable distribution
 - stable 55
- excessive function 261
- extended \mathcal{CBF} 80
 - integral representation 80
- extended Bernstein function
 - see* Bernstein function
- Feller transition function 379
- first passage time
 - and BO 289
 - and CE 289, 293
 - and GGC 289
 - and ME 293
 - generalized diffusion 285
- fractional power 129
 - Balakrishnan's formula 217
 - in \mathcal{BF} vii, 21, 33
 - in \mathcal{CBF} viii, 95
 - in \mathcal{CM} vii, 56
 - in \mathcal{S} viii, 19
 - in \mathcal{JBF} viii
 - of operators 216
- functional calculus 222
- functional inequality
 - Nash-type 233, 235, 236
 - Sobolev-type 233
 - super-Poincaré 239
 - weak Poincaré 240
- generalized diffusion 269
 - first passage time 285
 - inverse local time 270
 - local time 269
- generalized Gamma convolution 121
 - and first passage time 289
 - as vague closure 123
 - definition 121
 - stability 122
 - $GGC \subset BO$ 122
 - $GGC \subset SD$ 122
 - $GGC \not\subset ME$ 129
 - $GGC \not\subset SD_\infty$ 129
- harmonic function 262
- Heinz–Kato inequality 191
- Hellinger–Stone formula 182
- Helly's selection theorem 376

- Hirsch function 172
 - characterization 173
 - composition with \mathcal{CBF} 176
 - cone structure 173
 - definition 172
 - relation with \mathcal{P} 175
 - relation with \mathcal{SBF} 175
 - relation with \mathcal{S} 175
 - stability 173
- Hunt process 242, 269, 378
- inconsistency
 - unavoidable 1–411
- infinitely divisible
 - see* distribution, random variable
- inverse local time
 - and \mathcal{CBF} 270
 - generalized diffusion 270
- killed Brownian motion 260
- killed process 243
 - subordinate 246
- Kreĭn correspondence 284
 - examples 284–285
- Kreĭn representation problem 284
- Kreĭn–Milman representation
 - see* Choquet representation
- Laplace exponent 49
 - of distribution 127
 - relations among 128
- Laplace transform 1
 - uniqueness 2
- Lévy process 187, 212, 252
- Lévy triplet 22
- Lévy–Khintchine formula 46
- local time
 - generalized diffusion 269
- $\log\text{-}\mathcal{CM}$ 127
- $\log\text{-}\mathcal{S}$ 127
- $\log\text{-}$ completely monotone function
 - characterization 53
 - definition 53
 - relation with \mathcal{P} 64
- $\log\text{-}$ convex function 164
- $\log\text{-}$ convex sequence 164, 173
- Markov process 377
 - definition 377
 - generator 380
 - normal 377
 - quasi left-continuous 378
 - resolvent kernel 379
 - semigroup 379
 - strong 378
 - transition function 378
- Markov property 377
 - strong 378
- matrix monotone function 188
- measure
 - Lévy 22, 299
 - Nevanlinna–Pick 79
 - Pick 79
 - Stieltjes 75, 299
 - Thorin 111, 299
- mixture of exponential distributions
 - and first passage time 293
 - characterization 119
 - definition 118
 - stability 120
 - $ME \subset BO$ 120
 - $ME \not\subset GGC$ 129
 - $ME \not\subset SD$ 128
- monotone matrix function 188
- monotone of order n
 - see* $n\text{-}$ monotone function
- multiply monotone function
 - see* $n\text{-}$ monotone function
- $n\text{-}$ monotone function 8, 14
 - vs. $n\text{-}$ monotone sequence 14
 - integral representation 8–9
 - iterated differences 42
 - on \mathbb{R} 13
 - stability 12
- negative definite function 36, 212
 - in the sense of Schoenberg 46, 187, 212
- Nevanlinna factorization 85
- Nevanlinna–Pick function 77
 - integral representation 77
 - relation with \mathcal{CBF} 83
 - relation with \mathcal{S} 82–83
- Nevanlinna–Pick measure 79
- non-tangential limit 76, 90

- operator monotone function
 - definition 188
 - is \mathcal{CBF} 195
- p -monotone function
 - see* n -monotone function
- Phillips' theorem 204
- Pick function 77
 - integral representation 77
 - relation with \mathcal{CBF} 83
 - relation with \mathcal{S} 82–83
- Pick matrix 108
- portmanteau theorem 375
- positive definite function 35
 - conditionally 36
 - in the sense of Bochner 45
- potential 64, 97, 257
 - relation with \mathcal{H} 175
 - structure 97
- potential measure 63
 - λ -potential measure 64
- potential operator 257
 - λ -potential operator 270
- quasi everywhere 243, 381
- random variable
 - infinitely divisible 51, 54
 - stable 55
- representing measure
 - for \mathcal{BF} 22
 - for \mathcal{CBF} 75
 - for \mathcal{CM} 4
 - for \mathcal{PBF} 61
 - for \mathcal{S} 17
 - for \mathcal{TBf} 111
 - for n -monotone functions 8–9
 - for Nevanlinna–Pick fns 77, 79
- resolvent equation 179, 201, 379
- Revuz measure 382
- \mathcal{S} -self decomposable distribution
 - characterization 123
 - definition 123
 - is $\text{GGC} \cap \text{ME}$ 124
- Schoenberg's theorem 36
- Schoenberg–Bochner theorem 212
- self-adjoint operator
 - dissipative 180, 201
 - order 188
 - resolvent 179, 201
 - resolvent equation 179
 - resolvent estimate 201
 - spectral theorem 183
 - spectrum 179, 201
- self-decomposable distribution 55
 - completely (SD_∞) 60–61, 121
 - Lévy measure of 57
 - p -times (SD_p) 60
 - $\text{GGC} \not\subset \text{SD}_\infty$ 129
 - $\text{SD}_\infty \cap \text{CE}$ 125
 - $\text{SD}_\infty \cap \text{ME}$ 121
 - $\text{SD} \not\subset \text{BO}$ 127
 - $\text{SD} \not\subset \text{ME}$ 127
- semigroup
 - see also* convolution semigroup,
 - see also* subordinate semigroup
 - algebraic 35
 - C_0 -semigroup 200
 - contraction semigroup 200, 202
 - generator 200, 380
 - hypercontractive 241
 - intrinsically ultracontractive 257, 261
 - not hypercontractive 255
 - strongly continuous 200, 380
 - supercontractive 241
 - ultracontractive 233, 241, 247
- smooth measure 381
- special subordinator 257
- special Bernstein function 159, 164
 - cone structure 171
 - conjugate Lévy triplet 161–162
 - conjugate pair 159
 - counterexample 169–170
 - definition 159
 - potential density 161
 - relation with \mathcal{H} 175
 - stability 171
 - $\mathcal{SBf} \not\subset \mathcal{CBF}$ 169–170
- special functions 300–303
- special subordinator 159, 164
 - characterization 160
 - factorization of potential density 163, 258, 259
 - potential density 161

- spectral theorem 183, 189
- Stieltjes function 16
 - \mathcal{CBF} - \mathcal{S} 80
 - and string 273
 - cone structure 17
 - definition 16
 - extremal representation 18
 - log-convexity of \mathcal{S} 99
 - negative of 82–83
 - primitive 97
 - relation with \mathcal{CBF} 93, 96, 99
 - relation with \mathcal{H} 175
 - stability 17, 94, 120
- Stieltjes inversion formula 75
- Stieltjes transform
 - see* Stieltjes function
 - generalized 20, 116
- string 268
 - and Stieltjes function 273
 - characteristic function 274
 - definition 268
 - dual 284
 - length 268
- subordinate generator 203
 - \mathcal{CBF} -subordinator 220
 - domain 208, 219, 227–230
 - functional calculus 222
 - limits 230
 - moment inequality 209
 - operator core 204
 - Phillips' formula 205
 - spectrum 211
- subordinate process 65, 211, 244
 - killed 245
 - subordinate Brownian motion 91, 135, 216
- subordinate semigroup 65, 202–203
 - definition 203
 - in Hilbert space 203
 - of measures 65
- subordinator 49, 211
 - integral w.r.t. 131
 - killed 50
 - transition probabilities 49
- symmetric α -stable process 252
- table
 - of distributions 127–128
 - of Kreĭn correspondences 284
 - of Laplace exponents 127–128
 - of transformations of \mathcal{BF} 134, 136
- tangent functional 233
- Thorin–Bernstein function 109, 114
 - $\mathcal{TB}\mathcal{F}_\gamma$ 114
 - characterization 109, 112, 124
 - cone structure 113
 - definition 109, 114
 - extremal representation 114, 123
 - is in \mathcal{CBF} 109
 - relation with \mathcal{S} 113
 - Thorin measure 111
 - Thorin representation 111, 114, 122
- transformation of \mathcal{BF}
 - compositions 142, 143
 - continuity 140, 146, 149, 152
 - definition 132
 - domain 133
 - Lévy triplet of 136
 - not continuous 141
 - not one-to-one 142
 - overview 134, 136
 - parametrized families 133
 - range 133, 149, 152, 155, 156
 - \mathcal{PBF} 155, 156
- transition function 49, 378
- vague convergence 374
- vague integral 375
- weak convergence 374