

Index

The index should be used in conjunction with the Bibliography and the Index of Notation. Numbers following entries are page numbers which, if accompanied by (Pr. n), refer to Problem n on that page. Within the index we use the abbreviations ‘BM’ for Brownian motion and ‘LIL’ for law of iterated logarithm.

- absorbing point, 107
- acceptance–rejection method, 347
- action functional, 200, 207
- adapted, 46
- admissible filtration, 46, 59, 75, 219 (Pr. 1)
- arc-sine law
 - for largest zero, 74, 176
 - for the time in $(0, \infty)$, 122

- Banach–Steinhaus theorem, 385
- Blumenthal’s 0-1-law, 77, 128
- bounded variation, 136, 196, 220, 375, 383
- Box–Muller-method, 349
- Brownian bridge, 17 (Pr. 6)
- Brownian motion, 1, 4
 - canonical model, 38, 40
 - characteristic function, 7, 11–13
 - characterization, 10–13
 - Chung’s LIL, 187
 - Ciesielski’s construction, 23, 352
 - covariance, 8
 - dimension of graph, 166
 - dimension of image, 165, 169
 - dimension of zero set, 175
 - distribution of zeros, 176–178
 - Donsker’s construction, 34, 216, 352
 - with drift, 192, 274, 337
 - embedding of random variable, 211
 - excursion, 178
 - exit from annulus, 69
 - exit from ball, 70
 - exit from interval, 53, 69
 - exponential moments, 8
 - fast point, 184
 - first passage time, 54, 65
 - is Gaussian process, 9, 10
 - generator, 86–87, 94–96
 - geometric, 293
 - Hölder continuity, 138, 153, 158, 163
 - independent components, 13
 - interpolation, 28–30, 351
 - invariance principle, 34, 216, 352
 - Khintchine’s LIL, 182, 191
 - Kolmogorov’s construction, 34–35
 - large deviations, 202, 204
 - law of (\min, \max, B) , 71, 186
 - law of first passage time, 65
 - law of large numbers, 19 (Pr. 24), 181
 - law of running \min/\max , 65
 - level set, 172, 173
 - Lévy’s construction, 28–30, 351
 - Lévy’s modulus of continuity, 155
 - LIL, 182, 187, 191, 204
 - LIL fails, 184
 - local modulus of continuity, 184
 - local time, 263, 306
 - Markov property, 14, 59, 60
 - martingale characterization, 147, 272
 - maximal inequality, 48
 - modulus of non-differentiability, 189
 - moments, 8
 - nowhere differentiable, 153
 - nowhere monotone, 170
 - \mathcal{P} measurable, 77
 - projective reflection, 15, 41
 - quadratic variation, 137, 149 (Pr. 5)
 - record set, 175
 - reflected at b , 67
 - reflection, 14, 41
 - reflection principle, 64–65, 72
 - renewal, 14, 41
 - resolvent kernel, 93
 - return to zero, 70
 - scaling, 15, 41
 - slow point, 184
 - started at x , 4, 60
 - Strassen’s LIL, 191, 204
 - strong Markov property, 62–64, 66
 - strong variation, 159 (Pr. 5)
 - tail estimate, 155, 181
 - time inversion, 15, 41
 - transition density, 9, 34

- unique maximum, 171
- Wiener’s construction, 26
- zero set, 175
- Brownian scaling, 15, 41
- Burkholder inequalities, 285
- Burkholder–Davis–Gundy inequalities, 284, 288

- $\mathcal{C}_{(0)}$ is not measurable, 41
- Cameron–Martin
 - formula, 194, 197, 203
 - space, 192, 198
- canonical model, 38, 40
- canonical process, 42, 84
- carré-du-champ operator, 344 (Pr. 7)
- Cauchy problem, 118
- central limit method, 350
- Chapman–Kolmogorov equations, 62, 111 (Pr. 6)
- characteristic operator, 108
- Chung’s LIL, 187
- closed operator, 89
- completion, 75
- composition method, 348
- conditional expectation
 - under change of measure, 274
- cone condition, 128
 - flat cone condition, 135 (Pr. 9)
- consistent probability measures, 42
- covariance matrix, 7
- cylinder set, 39, 359

- diffusion coefficient/matrix, 3, 248, 297, 332, 333, 335, 336, 342
- diffusion process, 333, 334
 - backward equation, 335
 - construction by SDE, 342
 - defined by SDE, 339
 - forward equation, 336
 - generator, 333
- Dirichlet problem, 124, 133
- dissipative operator, 96
- Doléans–Dade exponential, 268, 270
- Donsker’s invariance principle, 34, 216, 352
- Doob
 - decomposition, 220
 - maximal inequality, 369
 - optional stopping theorem, 373
- Doob–Meyer decomposition, 237, 381
- Doss–Sussmann theorem, 328

- drift coefficient/vector, 248, 274, 297, 308, 333, 335, 336, 342
- Dynkin’s formula, 106
- Dynkin–Kinney criterion, 333

- energy, 163, 387
- Euler scheme, 354, 357
 - strong order, 355
 - weak order, 355

- Feller process, 85, 376
 - starting at x , 84
 - strong Markov property, 376
- Feller property, 82
 - strong Feller property, 82
- Feller’s minimum principle, 97
- Feynman–Kac formula, 121
- Fick’s law, 332
- finite variation, *see* bounded variation
- first entry time, *see* first hitting time
- first hitting time, 51
 - expected value, 53, 107, 125
 - of balls, 69, 70
 - properties, 57 (Pr. 12)
 - is stopping time, 51
- first passage time, 52, 54, 65
- Fokker–Planck equation, 336, 337
- formal adjoint operator, 336
- Friedrichs mollifier, 111 (Pr. 8), 262, 266 (Pr. 12)
- Frostman’s theorem, 163, 387, 389
- fundamental solution, 337
 - of diffusion equation, 337
 - existence, 338

- Gaussian
 - density (n-dimensional), 9
 - process, 8
 - random variable, 7
 - tail estimate, 155, 181
- generalized inverse, 282–283
- generator, 86
 - of Brownian motion, 86–87, 94–96
 - is closed operator, 89
 - is dissipative, 96
 - Dynkin’s generator, 108
 - Feller’s minimum principle, 97
 - is local operator, 109
 - no proper extension, 94
 - pointwise definition, 98

- positive maximum principle, 97
- of the solution of an SDE, 339
- weak generator, 98
- geometric BM, 293
- Girsanov transform, 198, 275, 289 (Pr. 5), 308
- Gronwall's lemma, 390

- Hölder continuity, 138
 - Hölder continuity of a BM, 138, 153, 158, 163
- Haar functions, 22
 - completeness, 23, 390
 - Haar–Fourier series, 23
- harmonic function, 125
 - and martingales, 126
 - maximum principle, 127
 - mean value property, 126
- Hausdorff dimension, 162
 - of a Brownian graph, 166
 - of a Brownian path, 165, 169
 - of Brownian zeros, 175
 - vs. capacitary dimension, 389
- Hausdorff measure, 160
- heat equation, 115, 116
- heuristics
 - for the Cauchy problem, 118
 - for Dirichlet's problem, 124–125
 - for Itô's formula, 250
 - for Kolmogorov's equations, 337–338
 - for an SDE, 290
 - for Strassen's LIL, 204
 - for Stratonovich SDE, 326
- Hille–Yosida theorem, 96
- holding point, 107

- independent increments (B1), 4
- infinitesimal generator, *see* generator
- instantaneous point, 107
- inverse transform method, 346
- Itô
 - differential *see* stoch. differential,
 - integral *see* stochastic integral,
 - isometry, 225, 228, 238, 244
 - process, 248, 249, 257
- Itô's formula
 - for BM^1 , 248
 - for BM^d , 258
 - for Itô process, 257
 - and local time, 264, 306
 - Tanaka's formula, 264, 306

- time-dependent, 261
- Kaufman's theorem, 169
- Khintchine's LIL, 182, 191
- Kolmogorov
 - backward equation, 335
 - consistency condition, 42
 - construction of BM, 34–35, 42–44
 - continuity criterion, 44, 150
 - existence theorem, 360
 - forward equation, 336
 - test for upper function, 184
- Kolmogorov–Chentsov thm, 44, 150

- Langevin SDE, 294
- Laplace operator, 48, 56 (Pr. 8), 97, 115, 118, 124
- large deviation
 - lower bound, 204
 - upper bound, 202
- Lebesgue's spine, 130–131
- level set, 172, 173
- Lévy's modulus of continuity, 155
- LIL, 182, 184, 186, 187, 191, 204
- linear growth condition, 300
- Lipschitz condition, 300
- local martingale, 243
 - is martingale, 270
- local operator, 108
 - and continuous paths, 109
 - is differential operator, 109
 - and diffusions, 333
- local time, 263, 306
- localizing sequence, 242, 243

- Markov process, 61
 - homogeneous Markov process, 61
 - starting at x , 61
 - strong Markov process, 64
- Markov property, 14, 59, 60
 - solution of SDE, 309, 310
- martingale, 46
 - backwards convergence theorem, 367
 - backwards martingale, 141, 142, 367
 - bounded variation, 375
 - characterization of BM, 147, 272
 - closure, 366
 - convergence theorem, 365
 - Doob–Meyer decomposition, 237, 381
 - embedding into BM, 283

- exit from interval, 69
- and generator, 49, 105, 114
- L^2 martingale, 220, 224
- local martingale, 243
- maximal inequality, 369
- norm in $\mathcal{M}_T^{L^2, c}$, 227, 240 (Pr. 3)
- quadratic variation, 146, 220–221, 381
- upcrossing, 364
- martingale representation
 - as time-changed BM, 283
 - if the BM given, 279
 - if the filtration is given, 280, 281
- martingale transform, 221
 - characterization, 223
- Maruyama’s drift transformation, 308
- mass distribution principle, 162
- maximum principle, 127
- mean value property, 126
- modification, 24
- modulus of continuity, 154–155
- Monte Carlo simulation, 358

- natural filtration, 46, 51
- Novikov condition, 55, 271

- optional stopping/sampling, 373
- Ornstein–Uhlenbeck process, 17 (Pr. 8), 294

- Paley–Wiener–Zygmund integral, 196–197, 241 (Pr. 13)
- path space, 40
- Poisson process, 159 (Pr. 1), 288 (Pr. 2)
- positive maximum principle, 97
- potential
 - linear potential, 102
 - logarithmic potential, 102
 - Newton potential, 102
- potential operator, *see* resolvent, 98, 102, 104
- progressive/progressively/ \mathcal{P}
 - measurable, 77, 241 (Pr. 18), 247 (Pr. 2)
 - σ -algebra, 234, 241 (Pr. 17)
- projective family, 42, 360
- projective limit, 360

- Q-Brownian motion, 13
- quadratic covariation, 222, 344 (Pr. 7)
- quadratic variation, 228, 248, 381
 - a. s. convergence, 139, 141
 - of BM, 137, 149 (Pr. 5)
 - L^2 -convergence, 137
 - of martingale, 146
 - strong q. v. diverges, 142
- random measure, 36 (Pr. 2)
- random variable
 - embedded into BM, 211
- random walk, 2–3, 33
 - Chung’s LIL, 186
 - embedded into BM, 213
 - Khinchine’s LIL, 214–215
- recurrence, 70
- reflection principle, 64–65, 72
- regular point, 128
 - criterion for, 128, 135 (Pr. 9)
- resolvent, 81, 90
 - resolvent equation, 91, 112 (Pr. 15)
- Riemann–Stieltjes integral, 197, 220, 383–385
- Riesz representation theorem, 385
- right-continuous filtration, 75

- sample path, 4
- sample value, 345
- Schauder functions, 22
 - completeness, 23, 390
- Schilder’s theorem, 201, 203
- semigroup, 81
 - conservative, 82, 85
 - contraction, 82
 - Feller, 82
 - Markov, 82
 - positive, 82
 - strong Feller, 82
 - strongly continuous, 82
 - sub-Markov, 82
- simple process, 224
 - closure, 236, 239
- simulation of BM, 351, 352
- singular point, 128
 - examples, 129–132
- Skorokhod’s embedding theorem, 212
- state space, 4
 - one-point compactification, 85, 333
- stationary increments (B2), 4
- stationary process, 18 (Pr. 20)
- stochastic differential, 249, 254
- stochastic differential equation, 290
 - continuity of solution, 313, 316
 - counterexamples, 304–308

- dependence on initial value, 313, 316, 317
- deterministic coefficients, 292
- differentiability of solution, 316
- of diffusion process, 339
- drift transformation, 297, 308
- examples, 292–296, 304–308, 327–328
- existence of solution, 302
- Feller continuity, 313
- for given generator, 342
- heuristics, 290
- homogeneous linear SDE, 295
- Langevin equation, 294
- linear growth condition, 300
- linear SDE, 295
- Lipschitz condition, 300
- local existence and uniqueness, 311
- localization, 310
- Markov property of solutions, 309
- measurability of solution, 304
- moment estimate, 313, 315
- no solution, 304–308
- numerical solution, 354, 357
- Picard iteration, 302
- solution map, 328
- solution of SDE, 290
- stability of solutions, 300
- Stratonovich SDE, 326
- transformation into linear SDE, 299
- transformation of SDE, 292, 296–297
- transition function of solutions, 310
- uniqueness of solutions, 302, 308
- variance transformation, 297
- weak solution, 306
- stochastic exponential, 268, 270
- stochastic integral, 228
 - Itô isometry, 228
 - localization, 229
 - is martingale, 228
 - maximal inequalities, 229
 - mean-square cont. integrand, 233
 - Riemann approximation, 233, 245
 - Stratonovich integral, 324
- stochastic integral (for $\mathcal{M}_T^{(2,c)}$), 238
 - Itô isometry, 238
 - localization, 238
 - is martingale, 238
 - maximal inequalities, 238
- stochastic integral (localized), 243
 - localization, 244
 - is martingale, 243
 - Riemann approximation, 245
 - stochastic integral (simple proc.), 224
 - Itô isometry, 225
 - localization, 226
 - is martingale, 225
 - stochastic process, 4
 - equivalent, 24
 - independence of two processes, 11
 - indistinguishable, 24
 - modification, 24
 - sample path, 4
 - simple, 224
 - state space, 4
 - stopping time, 50, 370
 - approximation from above, 372
 - Strassen’s LIL, 191
 - Stratonovich
 - differential equation, 326
 - Stratonovich integral, 324
 - vs. Itô integral, 325
 - strong Feller property, 82
 - strong Markov property, 62–64, 66, 376
 - strong order of convergence, 355
 - strong variation, 136, 220, 375, 383
 - Tanaka’s formula, 264, 306
 - trajectory, *see* sample path
 - transience, 70
 - transition kernel, 84
 - transition semigroup, *see* semigroup
 - true sample, 345
 - upcrossing estimate, 364
 - upper function, 184
 - usual conditions, 77
 - variation
 - *see also* bounded variation,
 - *see also* quadratic variation,
 - *see also* strong variation,
 - p -variation, 136
 - variation sum, 136, 383
 - Wald’s identity, 52, 69
 - exponential Wald identity, 55
 - weak order of convergence, 355
 - Wiener measure, 40, 192
 - Wiener process, *see* Brownian motion

Wiener space, 40, 192

Wiener–Fourier series, 26

Zaremba

– cone condition, 128

– deleted ball, 129

– needle, 132