# Brownian Motion — An Introduction to Stochastic Processes

de Gruyter Graduate, Berlin 2012, ISBN: 978-3-11-027889-7

by

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**Date: June 13, 2017.**

| PAGE, LINE | READS | SHOULD READ |
|---|---|---|
| **p. 6, Problem 6, (B3′)** | square integrable. | have variance 1. |
| **p. 12, l. 11 above** | $\mathsf{N}(0, t-s)$ | $\mathsf{N}(0, (t-s)I_d)$ |
| **p. 14, l. 4, l. 6 above** | BM$^d$ | BM$^1$ |
| **p. 15, (2.16), l. 12 above** | $\sigma\left(\bigcup_{n\geq 1}\bigcup_{0\leq t_1<\ldots<t_n<\infty}\sigma\big(X(t_j),\ldots,X(t_n)\big)\right)$ | $\sigma\left(\bigcup_{n\geq 1}\bigcup_{0\leq t_1<\ldots<t_n<\infty}\sigma\big(X(t_1),\ldots,X(t_n)\big)\right)$ |
| **p. 15, l. 15 above** | $(X(s_1),\ldots,X(s_n))\perp\!\!\!\perp(Y(t_1),\ldots,Y(t_m))$ | $(X(s_1),\ldots,X(s_m))\perp\!\!\!\perp(Y(t_1),\ldots,Y(t_n))$ |
| **p. 18, l. 7 above** | $e^{-\frac12\,\lvert\Sigma^\top\xi\rvert^2} = e^{-\frac12\,\langle\xi,\Sigma\Sigma^\top\xi\rangle}$ | $e^{-\frac12 t\,\lvert\Sigma^\top\xi\rvert^2} = e^{-\frac12 t\,\langle\xi,\Sigma\Sigma^\top\xi\rangle}$ |
| **p. 33, (3.6), l. 10 below** | $W(t,\omega) := \sum_{n=1}^\infty \frac{\sin(n\pi t)}{n}\,G_n(\omega)$ | $W(t,\omega) := tG_0(\omega) + \sqrt{2}\sum_{n=1}^\infty \frac{\sin(n\pi t)}{n\pi}\,G_n(\omega)$ |
| **p. 33, l. 4 below** | $W_N(t,\omega) := \sum_{n=1}^N \frac{\sin(n\pi t)}{n}\,G_n(\omega)$ | $W_N(t,\omega) := tG_0(\omega) + \sqrt{2}\sum_{n=1}^N \frac{\sin(n\pi t)}{n\pi}\,G_n(\omega)$ |
| **p. 39, Problem 5, l. 3 above** | In this case, the reverse implications hold, too. | Do the reverse implications hold, too? |
| **p. 39, Problem 6, l. 5 above** | $(B_q)_{q\in\mathbb{Q}}$ | $(B_q)_{q\in\mathbb{Q}\cap[0,\infty)}$ |
| **p. 41, l. 11 below** | $(v,w)\mapsto$ | $v\mapsto$ |
| **p. 42, l. 2 above** | $\omega\mapsto\rho(B(\cdot,\omega),w)$ | $\omega\mapsto\rho(B(\cdot,\omega),v)$ |
| **p. 49, l. 7 below** | $e^{\frac12\,\lvert\xi\rvert^2}$ | $e^{\frac t2\,\lvert\xi\rvert^2}$ |
| **p. 49, l. 4 below** | $M^\zeta(t) := e^{\langle\zeta,B(t)\rangle-\frac t2\,\lvert\zeta\rvert^2}$ | $M^\zeta(t) := \exp\left(\sum_{j=1}^d(\zeta_j B^j(t) - \frac t2\,\zeta_j^2)\right)$ |
| **p. 53, l. 2 below** | $\{\tau_U < t\}\in\mathscr{F}_t^X$ and $\{\tau_U\leq t\}\in\mathscr{F}_{t+}^X$ | $\{\tau_U < t\}\in\mathscr{F}_t^X$ and $\{\tau_U\leq t\}\in\mathscr{F}_{t+}^X$ for all $t\geq 0$, |
| **p. 54, l. 5 above** | $\bigcup_{\mathbb{Q}^+\ni r\leq t}$ | $\bigcup_{\mathbb{Q}^+\ni r<t}$ |
| **p. 54, l. 9 above** | $r\in(0,t]\cap\mathbb{Q}$ | $r\in(0,t)\cap\mathbb{Q}$ |
| **p. 54, l. 10 above** | $\tau_U(\omega)<r\leq t$ | $\tau_U(\omega)\leq r<t$ |
| **p. 60, Problem 9** | $a^2/2+b>0$ | $a^2/2+b=0$ |
| **p. 60, Problem 11 (c)** | $\tau^\circ_{A\cap C}=\max\{\tau^\circ_A,\tau^\circ_C\}$,<br>$\tau_{A\cap C}=\max\{\tau_A,\tau_C\}$ | $\tau^\circ_{A\cap C}\geq\max\{\tau^\circ_A,\tau^\circ_C\}$,<br>$\tau_{A\cap C}\geq\max\{\tau_A,\tau_C\}$ |
| **p. 66, l. 10 above** | (B1)/(5.1) | (B1)/5.1b) |
| **p. 68, l. 4 below** | and so | and so, for all $b>0$, |
| **p. 72, l. 8 below** | $\omega\mapsto B(\cdot\wedge\tau(\omega),\omega)-B(\tau(\omega),\omega)$ | $\omega\mapsto B(\cdot+\tau(\omega),\omega)-B(\tau(\omega),\omega)$ |
| **p. 72, l. 6 below** | $B(\cdot\wedge\tau)-B(\tau)\sim -B(\cdot\wedge\tau)-B(\tau)$ | $B(\cdot+\tau)-B(\tau)\sim -B(\cdot+\tau)+B(\tau)$ |
| **p. 74, l. 8 above** | $\mathbb{1}_{\mathbb{B}(0,r)}\leq\chi_{r,R}\leq\mathbb{1}_{\mathbb{B}(0,R)}$ | $\chi_{r,R}\lvert_{\mathbb{B}(0,R)\setminus\mathbb{B}(0,r)}\equiv 1$ and $\chi_{r,R}\lvert_{\mathbb{B}(0,r/2)}\equiv 0$ |
| **p. 75, l. 10 above** | (6.14) | (6.17) |
| **p. 82, l. 10 below** | $\mathscr{F}_0^B=\{\emptyset,\Omega\}$ | $\mathscr{F}_0^B\subset\sigma(\mathscr{N})$ |
| **p. 83, l. 4 below** | Let $(B_t)_t$ a BM$^1$ | Let $(B_t)_t$ a BM$^d$ |
| **p. 85, Problem 3, l. 5 above** | a $\sigma$-algebra | a Dynkin system |
| **p. 85, Problem 4** | $u(x)=u_n(x)\uparrow x$ | $u(x)=u_n(x)\to x$ |
| **p. 88, l. 14 above** | $\lim_{x\to y}u(B_t+x)=u(B_t+y)$ and $\lim_{\lvert x\rvert\to\infty}u(B_t+x)=0$. | $\lim_{x\to y}\mathbb{E}u(B_t+x)=\mathbb{E}u(B_t+y)$ and $\lim_{\lvert x\rvert\to\infty}\mathbb{E}u(B_t+x)=0$. |
| **p. 88, l. 7 above** | $\mathbb{P}^x(X_t\in\mathbb{R}^d)$ | $\mathbb{P}^x(B_t\in\mathbb{R}^d)$ |
| **p. 89, l. 3 above** | $\lvert y\rvert\geq 2$ | $\lvert y\rvert\geq 2R$ |

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| **p. 90, l. 5 below** | $\bigcap_{j=0}^{m}\dots$; $s_0 = 0 < s_1 < \dots$ | $\bigcap_{j=1}^{m}\dots$; $0 \le s_1 < \dots$ |
| **p. 93, l. 3 above** | $\partial u(x)$ $\partial^j\partial^k u(x+\theta B_t)$ | $\partial_j u(x)$ $\partial_j\partial_k u(x+\theta B_t)$ |
| **p. 95, l. 2 above** | $\mathcal{B}_b(\mathbb{R}^n)$ | $\mathcal{B}_b(\mathbb{R}^d)$ |
| **p. 95, l. 7 below** | This proves (7.11c) and (7.11b) follows now from part a). | This proves (7.11c) and (7.11b). |
| **p. 96, l. 11 above** | $P_{t-s-h}$ (three times) | $P_{t-s+h}$ (three times) |
| **p. 97, l. 5 above** | $\alpha U_\alpha$ is conservative: | $\alpha U_\alpha$ is conservative whenever $P_t$ is conservative: |
| **p. 98, l. 10/11 above** | Because of f) ... ...$(\beta-\alpha)U_\beta U_\alpha u$ | Let $\alpha,\beta > 0$ and $u \in \mathcal{B}_b$. Using the definition of $U_\alpha$ and Fubini's theorem we see that<br>$U_\alpha U_\beta u(x)$<br>$= \int_0^\infty\int_0^\infty e^{-\alpha t}e^{-\beta s}P_tP_su(x)\,ds\,dt$<br>$= \int_0^\infty\int_0^\infty e^{-\beta s}e^{-\alpha t}P_sP_tu(x)\,dt\,ds$<br>$= U_\beta U_\alpha u(x)$.<br>Thus, by part f),<br>$U_\alpha u - U_\beta u$<br>$= (\beta\,\mathrm{id} - A)U_\beta U_\alpha u - (\alpha\,\mathrm{id} - A)U_\alpha U_\beta u$<br>$= (\beta-\alpha)U_\beta U_\alpha u$. |
| **p. 98, l. 3 below** | $(\mathrm{id} - \alpha U_\alpha)u(x)$ | $(\mathrm{id} - \alpha U_\alpha)U_\alpha u(x)$ |
| **p. 98, l. 2 below** | $n!\,(\mathrm{id} - \alpha U_\alpha)u(x)$ | $n!\,(\mathrm{id} - \alpha U_\alpha)U_\alpha^n u(x)$ |
| **p. 99, (7.15) l. 8 above** | $e^{-\sqrt{2\alpha}\,y}$ | $e^{-\sqrt{2\alpha}\,\lvert y\rvert}$ |
| **p. 99, l. 9 above** | $\frac{\alpha}{2y^2}$ | $\frac{\alpha}{2\lvert y\rvert^2}$ |
| **p. 99, l. 9 above** | $\sqrt{2\alpha}y$ | $\sqrt{2\alpha}\,\lvert y\rvert$ |
| **p. 100, l. 7 above** | if $d \ge 1$ | if $d \ge 2$ |
| **p. 100, l. 10 above** | $(\mathfrak{A},\mathfrak{D}(\mathfrak{A}))$ extends | $(\mathfrak{A},\mathfrak{D}(\mathfrak{A}))$, $\mathfrak{D}(\mathfrak{A}) \subset \mathcal{C}_\infty(\mathbb{R}^d)$, extends |
| **p. 100, l. 12 below** | $u \in \mathfrak{D}(A)$ | $u \in \mathfrak{D}(\mathfrak{A})$ |
| **p. 102, l. 1 above** | Proposition 7.13 h) | Proposition 7.13 i) |
| **p. 105, l. 10 below** | $\mathbb{P}^x(\tau_x > t)$ | $\mathbb{P}^x(\tau_x \ge t)$ |
| **p. 106, l. 7 below** | $\mathbb{E}^{x_0}u(X_{\tau_V\wedge n}) - u(x)$ | $\mathbb{E}^{x_0}u(X_{\tau_V\wedge n}) - u(x_0)$ |
| **p. 110, Problem 5** | $(-n)\wedge u \vee n$ | $(-n)\vee u \wedge n$ |
| **p. 111, Problem 9** | $\exp(tA) := \sum_{j=0}^\infty A^j/j!$ | $\exp(tA) := \sum_{j=0}^\infty (tA)^j/j!$ |
| **p. 111, Problem 11** | for Example 7.20 | for Example 7.20. Let $(u_n)_{n\ge1} \subset \mathcal{C}_\infty^2(\mathbb{R})$ such that $\frac12 u_n''$ is a Cauchy sequence in $\mathcal{C}_\infty$ and that $u_n$ converges uniformly to $u$. |
| **p. 111, Problem 10 (a)** | $\frac{d}{ds}P_{t-s}T_s = P_{t-s}AT_s - P_{t-s}BT_s$ | $\frac{d}{ds}P_{t-s}T_s = -P_{t-s}AT_s + P_{t-s}BT_s$ |
| **p. 111, Problem 11 (b)** | $\int_0^x g(z)\,dz$ | $\int_0^x 2g(z)\,dz$ |
| **p. 111, Problem 11 (b)** | $u_n(x)$ converges uniformly to $\int_0^x 2g(z)\,dt + c'$ | $u_n'(x)$ converges uniformly to $\int_0^x 2g(z)\,dz + c'$ |
| **p. 111, Problem 11 (b)** | $c' = \int_{-\infty}^0 g(z)\,dz$ | $c' = \int_{-\infty}^0 2g(z)\,dz$ |
| **p. 112, Problem 13** | $\frac{d^n}{d\alpha^n}(\alpha U_\alpha)f(x)$<br>$= n!(-1)^{n+1}(\mathrm{id} - \alpha U_\alpha)f(x)$ | $\frac{d^n}{d\alpha^n}(\alpha U_\alpha)f(x)$<br>$= n!\,(-1)^{n+1}(\mathrm{id} - \alpha U_\alpha)U_\alpha^n f(x)$ |
| **p. 113, l. 13 above** | $L = \sum_{j,k=1}^d \partial_j$ | $L = \sum_{j,k=1}^d \partial_j\partial_k$ |
| **p. 115, l. 8 below** | Proposition 7.4 g) | Proposition 7.3 g) |
| **p. 116, l. 5 above** | $\chi$ | $\chi_n$ |
| **p. 116, l. 6 above** | $\chi_n\big\rvert_{[1/n,n]\times\mathbb{B}(0,n)} \equiv 1$ | $\chi_n\big\rvert_{[1/n,t]\times\mathbb{B}(0,n)} \equiv 1$ |
| **p. 116, l. 8 below** | $f(x) \le$ | $\lvert f(x)\rvert \le$ |
| **p. 118, l. 2 above** | (8.7) satisfying | (8.7) with $g = g(x)$ satisfying |
| **p. 118, l. 5 above** | $\mathcal{C}_b^{1,2}$ | $\mathcal{C}^{1,2}$ |
| **p. 119, l. 9 below** | $N_s^w$ | $N^w$ |
| **p. 121, l. 7, 8 above** | $T_tf$ (twice) | $T_tu$ (twice) |
| **p. 123, l. 3 above** | $\frac{e^{-\lambda t}}{\sqrt{\pi r(t-r)}}$ | $\frac{e^{-\lambda t}}{\sqrt{\pi(t-r)}}$ |
| **p. 126, l. 10 above** | $\mathbb{P}^0(B_\sigma \in dz)$ | $\mathbb{P}^0(B_\tau \in dz)$, where $\tau = \tau_{\mathbb{B}^c(0,r)}$. |

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| **p. 126, l. 11 above** | $\mathbb{P}^0(B_\sigma \in dz)$ | $\mathbb{P}^0(B_\tau \in dz)$ |
| **p. 126, l. 14 above** | $\phi\vert_{[\delta^2,\infty)}$ | $\phi_\delta\vert_{[\delta^2,\infty)}$ |
| **p. 127, l. 2, 3 above** | $+\int_0^{t\wedge\sigma}$ (twice) | $-\int_0^{t\wedge\sigma}$ (twice) |
| **p. 127, l. 4 above** | $\mathbb{B}(x,\delta)$ | $\mathbb{B}(x_0,\delta)$ |
| **p. 127, l. 3 below** | $u\vert_{\partial B_r} \equiv u(x_0)$ | $u\vert_{\partial\mathbb{B}(x_0,r)} \equiv u(x_0)$ |
| **p. 129, l. 8 above** | therefore, $\min_{0\le j\le n}\tau_{V_j} \le \tau_{\mathbb{B}(x_0,\epsilon)}$, i.e. | thus, $\min_{0\le j\le n}\tau_{V_j} \le \tau_{\mathbb{B}(x_0,\epsilon)\setminus\{x_0\}}$, i.e. |
| **p. 129, l. 9 above** | $\tau_{\mathbb{B}(x_0,\epsilon)} > 0$ | $\tau_{\mathbb{B}(x_0,\epsilon)\setminus\{x_0\}} > 0$ |
| **p. 130, Figure 8.4** | $\mathbb{B}(1,0)$, $\mathbb{B}(2,c_n e_1)$ | $\mathbb{B}(0,1)$, $\mathbb{B}(c_n e_1, 2)$ |
| **p. 131, l. 9 below** | $\tau_{\mathbb{B}(0,1)}$ | $\tau_{\mathbb{B}^c(0,1)}$ |
| **p. 132, l. 15 above** | $t\in[0,\epsilon]$ | $t\in(0,\epsilon]$ |
| **p. 132, l. 12 below** | $e_1=(1,0,\dots,0)\in\mathbb{R}^d$ | $e_2=(0,1,0,\dots,0)\in\mathbb{R}^d$ |
| **p. 132, l. 11 below** | $[0,\infty)e_1$ | $[0,\infty)e_2$ |
| **p. 133, l. 5 below** | $X_\tau$ | $B_\tau$ |
| **p. 136, Problem 4** | $P_t g - g = \frac{d}{dt}\int_0^t P_s g\,ds$ | $P_t g - g = \frac{d}{dt}\int_0^t P_s g\,ds - g$ |
| **p. 136, Problem 7** | multiply it with a smooth cut-off function $\mathcal{C}_c^2(\mathbb{R}^d)$, $u\vert_{\mathbb{B}(0,r)}\equiv 1$ | multiply it with a smooth cut-off function $\chi\in\mathcal{C}_c^2(\mathbb{R}^d)$, $\chi\vert_{\mathbb{B}(0,r)}\equiv 1$ |
| **p. 136, Prob. 9, l. 6 below** | Example 8.12 d) | Example 8.15 e) |
| **p. 138, l. 8 below** | mean zero | mean $t_j - t_{j-1}$ |
| **p. 139, l. 7 below** | $S_2^\Pi(B(\cdot,\omega),\Pi)$ | $S_2^\Pi(B(\cdot,\omega),[a,b])$ |
| **p. 141, l. 2 above** | $\big(B(t_j)-B(t_{j-1})\big)^2$ | $B(t_j)-B(t_{j-1})$ |
| **p. 145, l. 12 below** | $\Pi_{k+1}\subset\Pi_k$ | $\Pi_{k+1}\supset\Pi_k$ |
| **p. 147, l. 9 above** | $s\le t_j<t_k<t$ | $s\le t_j<t_k\le t$ |
| **p. 149, l. 1–7 below** | Using the elementary estimate<br>$\Big\lvert\prod_{j=1}^n a_j - \prod_{j=1}^n b_j\Big\rvert \le \sum_{j=1}^n \lvert a_j-b_j\rvert$<br>for all $a_j,b_j\in\mathbb{C}$, $\lvert a_j\rvert,\lvert b_j\rvert\le 1$, we find<br>$\Big\lvert\mathbb{E}\Big(e^{i\xi(X_t-X_s)} - e^{-\frac12\xi^2(t-s)} \,\Big\vert\, \mathscr{F}_s\Big)\Big\rvert$<br>$\le \mathbb{E}\left(\left\lvert e^{i\xi(X_t-X_s)} - e^{-\frac12\xi^2(t-s)}\right\rvert \,\Big\vert\, \mathscr{F}_s\right)$<br>$= \mathbb{E}\Bigg(\Bigg\lvert\prod_{j=1}^n e^{i\xi(X_{t_j}-X_{t_{j-1}})}$<br>$\qquad - \prod_{j=1}^n e^{-\frac12\xi^2(t_j-t_{j-1})}\Bigg\rvert \,\Bigg\vert\, \mathscr{F}_s\Bigg)$<br>$\le \sum_{j=1}^n \mathbb{E}\left(\left\lvert e^{i\xi(X_{t_j}-X_{t_{j-1}})} - e^{-\frac12\xi^2(t_j-t_{j-1})}\right\rvert \,\Big\vert\, \mathscr{F}_s\right).$ | Using the elementary estimate<br>$\lvert aA-bB\rvert \le \lvert a-b\rvert\cdot\lvert A\rvert + \lvert b\rvert\cdot\lvert A-B\rvert$<br>where<br>$a=\mathbb{E}\Big(e^{i\xi(X_t-X_{t_{n-1}})} \,\Big\vert\, \mathscr{F}_{t_{n-1}}\Big)$,<br>$A=e^{i\xi(X_{t_{n-1}}-X_s)}$,<br>$b=e^{-\frac12\xi^2(t-t_{n-1})}$,<br>$B=e^{-\frac12\xi^2(t_{n-1}-s)}$,<br>yields, because of the tower property,<br>$\Big\lvert\mathbb{E}\Big(e^{i\xi(X_t-X_s)} - e^{-\frac12\xi^2(t-s)} \,\Big\vert\, \mathscr{F}_s\Big)\Big\rvert$<br>$\le \mathbb{E}\Big(\Big\lvert\mathbb{E}\Big\{e^{i\xi(X_t-X_{t_{n-1}})}$<br>$\qquad - e^{-\frac12\xi^2(t-t_{n-1})} \,\Big\vert\, \mathscr{F}_{t_{n-1}}\Big\}\Big\rvert \,\Big\vert\, \mathscr{F}_s\Big)$<br>$\qquad + \Big\lvert\mathbb{E}\Big(e^{i\xi(X_{t_{n-1}}-X_s)} - e^{-\frac12\xi^2(t_{n-1}-s)} \,\Big\vert\, \mathscr{F}_s\Big)\Big\rvert$<br>$\le\cdots\le$<br>$\le \sum_{j=1}^n \mathbb{E}\Big(\Big\lvert\mathbb{E}\Big\{e^{i\xi(X_{t_j}-X_{t_{j-1}})}$<br>$\qquad - e^{-\frac12\xi^2(t_j-t_{j-1})} \,\Big\vert\, \mathscr{F}_{t_{j-1}}\Big\}\Big\rvert \,\Big\vert\, \mathscr{F}_s\Big).$ |
| **p. 151, Problem 6 (b)** | $\{\lvert B((k+1)2^{-n}) - B(k2^{-n})\rvert > c\sqrt{n}\,2^{-n}\}$ | $\Big\{\lvert B((k+1)2^{-n}) - B(k2^{-n})\rvert > c\sqrt{n\,2^{-n}}\Big\}$ |
| **p. 151, Problem 6 (b)** | $\bigcup_{k=1}^{2^n} A_{k,n}$ (twice) | $\bigcup_{k=0}^{2^n-1} A_{k,n}$ (twice) |
| **p. 155, l. 12 above** | $c_n$ | $c_k$ |
| **p. 156, l. 3 below** | $\mathbb{P}\Big(\Big\{\Big\lvert B\big(\frac{i}{k}\big) - B\big(\frac{i-1}{k}\big)\Big\rvert \le \frac{7L}{k}\Big\}\Big)^3$ | $\prod_{i=j+1}^{j+3}\mathbb{P}\Big(\Big\{\Big\lvert B\big(\frac{i}{k}\big) - B\big(\frac{i-1}{k}\big)\Big\rvert \le \frac{7L}{k}\Big\}\Big)$ |
| **p. 160, l. 7 below** | $m(\epsilon,\omega)$ | $m(\delta,\omega)$ |
| **p. 161, l. 5 below** | nowhere | not |
| **p. 161, l. 1 below** | at no point $t\in[0,1)$ Hölder | at some $t\in[0,1)$ not Hölder |
| **p. 162, Problem 2** | $\le n\,\max_{1\le j\le n}\lvert x_j\rvert$ | $\le n^{1/p}\,\max_{1\le j\le n}\lvert x_j\rvert$ |
| **p. 167, l. 9 above** | for all | for (Lebesgue) almost all |
| **p. 172, Problem 2** | (A.14) | (A.13) |

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| **p. 172, Problem 2** | to show that | to show that for all $x, \xi > 0$ |
| **p. 172, Problem 4** | $\tau := \inf\left\{t \geq 0 \,:\, B_t = b\sqrt{a+t}\right\}$ | $\tau := \inf\left\{t \geq 0 \,:\, \lvert B_t\rvert = b\sqrt{a+t}\right\}$ |
| **p. 178, l. 2 below** | $2\left[e^{c\lvert B(1)\rvert}\right]$ | $2\mathbb{E}\left[e^{c\lvert B(1)\rvert}\right]$ |
| **p. 179, l. 2 above** | $F(\Pi_n w)$ | $F(w)$ |
| **p. 179, l. 7 above** | $w_0'(t)$ | $w_0(t)$ |
| **p. 179, l. 6 below** | $dw(t)$ | $dw(s)$ |
| **p. 181, l. 7 above** | $B_t(U^{-1}\omega)$ | $U^{-1}B_t(\omega)$ |
| **p. 182, l. 10 below** | $c \geq 0$ | $r \geq 0$ |
| **p. 182, l. 2 below** | $\mathbb{P}(d(\epsilon B, \Phi(r_0)) \geq \delta)$ | $\mathbb{P}(d(\epsilon B, \Phi(r)) \geq \delta)$ |
| **p. 188, l. 7 below** | $= \mathbb{P}(\gamma B \in F)$ | $\leq \mathbb{P}(\gamma B \in F)$ |
| **p. 189, l. 9 above** | $L(\omega)$ | $\mathcal{L}(\omega)$ |
| **p. 189, l. 11 below** | $c > q^{-1}$ | $c \geq q^{-1}$ |
| **p. 189, l. 2 below** | $(s'_{j(n)})_{j\geq 1}$ | $(s'_{j(n)})_{n\geq 1}$ |
| **p. 189, l. 1 below** | $(Z_{s'_{j(n)}}(\cdot,\omega))_{j\geq 1}$ | $(Z_{s'_{j(n)}}(\cdot,\omega))_{n\geq 1}$ |
| **p. 191, l. 1 above** | $q > 0$ | $q > 1$ |
| **p. 191, l. 3 above** | $w(1),$ | $w(1) - w(\frac{1}{q}),$ |
| **p. 191, l. 5 above** | $I(v) = \int \ldots = \int \ldots \leq \int \ldots \leq r < \frac{1}{2}$ | $I(v) = \frac{1}{2}\int \ldots = \frac{1}{2}\int \ldots \leq \frac{1}{2}\int \ldots \leq r < \frac{1}{2}$ |
| **p. 191, l. 5 above** | any $q > 1$ | large $q > 1$ |
| **p. 191, l. 7 above** | $\overline{\lim}$ | $\underline{\lim}$ |
| **p. 191, l. 7 above** | $\frac{B(ts_n)-B(s_{n-1})}{\sqrt{2\log\log s_n}}$ | $\frac{B(ts_n)-B(s_{n-1})}{\sqrt{2s_n\log\log s_n}}$ |
| **p. 191, l. 9 above** | $\frac{\sqrt{r}}{\sqrt{q}}$ | $\frac{\sqrt{2r}}{\sqrt{q}}$ |
| **p. 191, l. 15 below** | $\lvert W(ts_n)\rvert$ | $\lvert B(ts_n)\rvert$ |
| **p. 192, Prob. 4(c), l. 10 below** | $\pi$ | $\phi$ |
| **p. 195, l. 11 below** | $B_{\tau_{U,W}}$ | $B_{\tau^\circ_{(U,W)^c}}$ |
| **p. 195, l. 6 below** | $B_{\tau_{U,W}}$ | $B_{\tau^\circ_{(U,W)^c}}$ |
| **p. 197, l. 7 above** | variance $\sigma$ | variance $\sigma^2$ |
| **p. 200, l. 5 below** | $\leq s \leq$ | $\leq s\cdot n \leq$ |
| **p. 200, l. 6 below** | $B^n(\tau_k)$ and $B^n(\tau_{k+1})$ | $B^n(\tau_k/n)$ and $B^n(\tau_{k+1}/n)$ |
| **p. 200, l. 2 below** | $B^n(\tau_k)$ and $B^n(\tau_{k+1})$ | $B^n(\tau_k/n)$ and $B^n(\tau_{k+1}/n)$ |
| **p. 206, l. 8 below** | $I \in \mathcal{M}^2$ | $I \in \mathcal{M}^2$ with $I_0 = 0$ |
| **p. 206, l. 4 below** | Theorem 14.4 c) | Lemma 14.2 |
| **p. 206, l. 2 below** | adding | subtracting |
| **p. 207, l. 4 below** | $j = 0, \ldots, n$ | $j = 0, \ldots, n-1$ |
| **p. 215, l. 6 below** | Theorem 14.9 f) | Theorem 14.9 c) |
| **p. 221, l. 11 above** | predictable | progressive |
| **p. 224, l. 2 above** | $\{\tau_j \leq t\} = \{A_t > jT/n\}$ | $\{\tau_j \leq t\} = \{A_t \geq jT/n\}$ |
| **p. 224, l. 8 above** | $\sigma \leq \tau$ | $\sigma \leq \tau \leq T$ |
| **p. 224, l. 11 above** | $\sum_{k=1}^{m\lfloor T\rfloor}$ | $\sum_{k=1}^{\lfloor mT\rfloor}$ |
| **p. 224, l. 11 below** | $\{\xi \in B\} \cap \{\sigma \leq (k-1)/m\} \cap \{(k-1)/m < \tau\}$ | $\{\xi \in B\} \cap \{\sigma < (k-1)/m\} \cap \{(k-1)/m \leq \tau\}$ |
| **p. 225, Problem 8** | cf. Theorem A.21 — $\langle f\rangle \bullet B_t$ | cf. Proposition A.22 — $\langle f \bullet B\rangle_t$ |
| **p. 226, Problem 12** | $(B_{s_j} - B_{s_{j-1}})$ | $(B_{s_j\wedge t} - B_{s_{j-1}\wedge t})$ |
| **p. 228, l. 5 above** | $\int_0^t \lvert f(s,\cdot)\rvert^2\,ds > n$ | $\int_0^t \lvert f(s,\cdot)\rvert^2\,ds \geq n$ |
| **p. 228, l. 7 above** | $\int_0^t \lvert f(s,\cdot)\rvert^2\,ds \leq n$ | $\int_0^t \lvert f(s,\cdot)\rvert^2\,ds < n$ |
| **p. 230, l. 11 above** | $\mathbb{1}_A$ (twice) | $\mathbb{1}_F$ (twice) |
| **p. 234, l. 8 below** | $\ldots\inf\left\{t \geq 0 \,:\, \int_0^t \lvert\sigma_{jk}(s,\omega)\rvert^2\,ds > n\right\}$ | $\ldots\inf\left\{t \geq 0 \,:\, \int_0^t \lvert\sigma_{jk}(s,\omega)\rvert^2\,ds \geq n\right\}$ |
| **p. 239, l. 3 below** | $X_t^\Pi = \int_0^t \sigma^\Pi(s)\,dB_s + \int_0^t b^\Pi(s)\,ds$ | $X_t^\Pi = X_0 + \int_0^t \sigma^\Pi(s)\,dB_s + \int_0^t b^\Pi(s)\,ds$ |

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| PAGE, LINE | READS | SHOULD READ |
|---|---|---|
| **p. 242, l. 7 above** | $L^2(\mathbb{P})\text{-}\lim_{\lvert\Pi\rvert\to 0} \mathsf{J}_{21} = \int_0^t f''(X_s)\sigma(s)\,ds$ | $L^2(\mathbb{P})\text{-}\lim_{\lvert\Pi\rvert\to 0} \mathsf{J}_{21} = \int_0^t f''(X_s)\sigma^2(s)\,ds$ |
| **p. 244, l. 6 below** | $\lim_{n\to\infty}$ | $\lim_{\epsilon\to 0}$ |
| **p. 245, l. 1 above** | $(2\epsilon)^{-1}$ | $\epsilon^{-1}$ |
| **p. 245, l. 3 above** | $\dfrac{1}{2\epsilon}$ | $\dfrac{1}{\epsilon}$ |
| **p. 245, l. 8 above** | $d$-dimensional | one-dimensional |
| **p. 246, Problem 3** | $\int_0^t f(s, B_s)\,dB_s$ | $\int_0^t \frac{\partial f}{\partial x}(s, B_s)\,dB_s$ |
| **p. 247, Problem 7** | $u_x^2 + v_y^2 = 1$ | $u_x^2 + u_y^2 = 1$ |
| **p. 249, l. 3 above** | a martingale | a local martingale |
| **p. 250, l. 2 below** | $M^{\tau_n} = \exp(X_t^{\tau_n})$ | $M^{\tau_n} = \exp(X^{\tau_n})$ |
| **p. 254, (17.8) l. 9 below** | $\mathbb{P}(F) +$ | $\mathbb{P}(F) -$ |
| **p. 259, l. 1 below** | $M_T = \mathbb{E}M_T + \dots$ | $M = \mathbb{E}M + \dots$ |
| **p. 262, l. 2 below** | $s \le T$ | $t \le T$ |
| **p. 264, l. 3 above** | $a(\tau(s)) = s$ | $a(\tau(s)) = s$ for all $s \in a([0,\infty))$ |
| **p. 265, l. 2 above** | $\sup\{u > s \dots$ | $\sup\{u \ge s \dots$ |
| **p. 267, l. 10 below** | $p\int_0^t \lvert X_s\rvert^{p-1}\,dX_s$ | $p\int_0^t \operatorname{sgn}(X_s)\lvert X_s\rvert^{p-1}\,dX_s$ |
| **p. 269, l. 11 below** | $= \mathbb{P}(\cdots)$ | $\le \mathbb{P}(\cdots)$ |
| **p. 269, l. 1 below** | $\sigma > T$ | $\sigma \ge T$ |
| **p. 270, (17.18) l. 9 above** | $\int_0^T f(s)\,dB_s$ | $\int_0^t f(s)\,dB_s$ |
| **p. 271, Problem 10** | $f \in \mathsf{BV}[0,\infty)$ | $f : [0,\infty) \to \mathbb{R}$ be absolutely continuous |
| **p. 271, Problem 11** | (17.17) | (17.18) |
| **p. 276, l. 3 above** | $\int_0^t (\gamma(s) + \delta(s)X_s)\delta(s)X_s^\circ\,ds$ | $(\gamma(t) + \delta(t)X_t)\delta(t)X_t^\circ\,dt$ |
| **p. 284, l. 13 above** | then $(t, x, \omega) \mapsto X_t^x(\omega)$ | then $(s, x, \omega) \mapsto X_s^x(\omega)$, $(s, x) \in [0,t] \times \mathbb{R}^n$ is for every $t \ge 0$ |
| **p. 284, l. 3 below** | $\sigma(s_j, X_n^x(s_j))$ | $\sigma(s_{j-1}, X_n^x(s_{j-1}))$ |
| **p. 287, l. 6 above** | $\inf\{t \ge 0 : \lvert X_t^j - \xi\rvert \ge R\} \wedge T$ | $\inf\{t \ge 0 : \lvert X_t^j\rvert \ge R\} \wedge T$ |
| **p. 287, l. 1 below** | for all $x, y \in \mathbb{R}^n$ | for all $x \in \mathbb{R}^n$ |
| **p. 288, l. 2 below** | Theorem 18.9 | Theorem 18.11 |
| **p. 288, l. 1 below** | $\mathbb{E}\big[\lvert\chi_R(\xi)\xi\rvert^2\big]$ | $\mathbb{E}\big[(1 + \lvert\chi_R(\xi)\xi\rvert)^2\big]$ |
| **p. 290, l. 5 above** | $\big(\lvert x\rvert^2 + \lvert y\rvert^2\big)$ | $\big((1+\lvert x\rvert)^2 + (1+\lvert y\rvert)^2\big)$ |
| **p. 290, l. 6 above** | $\lvert x\rvert^2$ | $(1+\lvert x\rvert)^2$ |
| **p. 292, l. 4 above** | $x \in \mathbb{R}^d$ | $x \in \mathbb{R}^n$ |
| **p. 296, Problem 4 (e)** | $U_t := e^{-\beta t}B(e^{2\beta t} - 1)$ | $U_t := \frac{\sigma}{\sqrt{2\beta}}\,e^{-\beta t}B(e^{2\beta t} - 1)$ |
| **p. 301, l. 7 below** | $x_i$ | $x_j$ |
| **p. 303, l. 4 below** | $\partial_j\partial_k$ | $\partial_i\partial_j$ |
| **p. 311, l. 10 below** | $Lf(X_r)$ | $Lu(X_r)$ |
| **p. 329, l. 7 above** | $Z = \pi_J(B)$ | $Z = \pi_J^{-1}(B)$ |
| **p. 333, l. 11 below** | from $E$ | from $\mathbb{R}^d$ |
| **p. 338, l. 9 below** | $t \le t_0$ | $t \le t_1$ |
| **p. 342, l. 12 below** | $\inf_j \tau_j \le t + \frac{1}{n}$ | $\inf_j \tau_j < t + \frac{1}{n}$ |
| **p. 345, l. 5 below** | which proves (A.17) | which proves (A.16) |
| **p. 347, l. 9 above** | a submartingale | a submartingale with continuous paths |
| **p. 349, l. 6 below** | $F \cap \{\sigma_j = k/2^j\} \in \mathscr{F}_{\sigma+}$ | $F \cap \{\sigma_j = k/2^j\} \in \mathscr{F}_{k/2^j}$ |
| **p. 349, l. 4 below** | $\mathbb{E}^x[\mathbb{E}(\cdots)]$ | $\mathbb{E}^x[\mathbb{E}^x(\cdots)]$ |
| **p. 350, l. 2 above** | $\sum_{k=0}^\infty$ | $\sum_{k=1}^\infty$ |
| **p. 350, l. 11 above** | linear maps | continuous linear maps |
| **p. 351, l. 11 above** | $M^{\Pi,t} = (M_{t_j}^t, \mathscr{F}_{t_j\wedge t})_{t_{j-1},t_j\in\Pi}$ | $M^{\Pi,t} = (M_{t_j}^t, \mathscr{F}_{t_j\wedge t})_{t_j\in\Pi}$ |
| **p. 351, l. 9 below** | $\sum_{k=1}^n$ | $\sum_{j=1}^n$ |

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| PAGE, LINE | READS | SHOULD READ |
|---|---|---|
| **p. 351, l. 1 below** | $M_T^2 - M_0^2$ | $M_t^2 - M_0^2$ |
| **p. 352, l. 3 above** | $M_{\wedge t_{j-1}}$ | $M_{t \wedge t_{j-1}}$ |
| **p. 352, l. 8 above** | $2N_T^{\Pi,t}$ | $-2N_T^{\Pi,t}$ |
| **p. 352, l. 6 below** | $\lvert\Pi\rvert \wedge \lvert\Pi'\rvert$ | $\lvert\Pi\rvert \vee \lvert\Pi'\rvert$ |
| **p. 353, l. 6 below** | $t \in [0,t]$ | $t \in [0,T]$ |
| **p. 356, l. 4 above** | $q \in [0, T-q]$ | $t \in [0, T-q]$ |
| **p. 356, l. 7 above** | $\langle M^\tau \rangle_t \equiv \langle M \rangle_q$ | $\langle M \rangle_{t\wedge\tau+q} \equiv \langle M \rangle_q$ |
| **p. 356, l. 6 below** | $S_p^\Pi(f;t)$ | $S_1^\Pi(f;[a,b])$ |
| **p. 360, l. 3 above** | $\mathrm{VAR}_1(f;[a,b])$ | $\mathrm{VAR}_1(\alpha;[a,b])$ |

We would like to thank the following readers for their comments and their contributions to this list:
*Georg Berschneider (Dresden/Paderborn), Julian Hollender (Dresden), Johannes Huhn (Dresden), Franziska Kühn (Dresden), Felix Lindner (Dresden), Klaus Ritter (Kaiserslautern), Daniel Tillich (Dresden), Luciano Tubaro (Trento)*