René L. Schilling: Measure, Integral, Probability & Processes. 2nd edition.
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Preface

The material is an honest rendering of my lecture notes of the courses »Measure and Integration« (3 contact hours §§1–20), »Introduction to probability« (4 contact hours, §§21–35, 39–41), »Further probability theory: Discrete random processes« (3–4 contact hours, §§36, 52–57 and a selection from either §§58–61 or §§48–51) and »Probability with martingales« (3–4 contact hours, §§36–38, 42–47, 62–68) at TU Dresden. It is an introduction to measure and integration – suitable both for analysts and probabilists – and to probability theory & random processes up to the construction and first properties of Brownian motion.

The text is suitable for BSc students who have had a rigorous course in linear algebra and ϵ-δ-analysis. Some basic knowledge of functional analysis is helpful. The textbooks by Lang [26] (for linear algebra) and Rudin [38] (for analysis), [39, Chapters 4, 5] (for functional analysis) should be more than sufficient. For additional reference, I recommend Alt [2] (functional analysis) and the wonderful first chapter *Operator theory in finite-dimensional vector spaces* in Kato [22].

These notes contain the bare necessities. A more thorough treatment and plenty of exercises can be found in my books *Measures, Integrals and Martingales* [MIMS] and *Counterexamples in Measure and Integration* (with F. Kühn) [CEX] and my German-language textbooks *Maß und Integral* [MI], *Wahrscheinlichkeit* [WT] and *Martingale und Prozesse* [MaPs]. More on Brownian motion and stochastic (Itô) calculus is in *Brownian Motion. A Guide to Random Processes and Stochastic Calculus* [BM]. I tried to be as close as possible to the original lectures. Some essential material, which is usually set as (guided) exercise in the problem classes, is added as »starred sections« or »starred items« like **Theorem***. Handouts are either set in small print (if they appear in the running text) or contained in chapter appendices.

The subtitle »the theoretical minimum« alludes to the physicist Lev Landau, who *developed a comprehensive exam called the "Theoretical Minimum" which students were expected to pass before admission to the school* `https://en.wikipedia.org/wiki/Kharkiv_Theoretical_Physics_School` (accessed 26/Sep/2025).

For the second edition, I have carefully revised the text, removing quite a few blunders and misprints, and streamlining the presentation. Some previously »hidden« (and some new!) subsections appear now in the list of contents, to simplify the use of the book. In Chapter IV (§ 18) I included a short, fit-for-the-classroom proof of Jacobi's transformation theorem for Lebesgue integrals. More serious changes took place in Chapter XI on Markov chains, where I added the classical proof of the ergodic theorem (§ 57) as well as an introduction to stochastic simulation (§ 58) and Markov chain Monte Carlo (MCMC) methods (§ 59). On my web-page `https://www.motapa.de` you will find corrections and further updates.

I was quite surprised that the first edition of these notes was not only used locally in Dresden, but many more copies were sold all over the world and quite a few readers sent me encouraging e-mails or left positive comments on AMAZON. Thank you very much! I hope that these notes help you to enjoy maths as much as I do.

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